

THE DESIGN OF THE PYRAMID OF KHAEFRE

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In recent articles¹ the writer has described the geometrical basis for the passage-system in the Pyramid of Khufu, showing the great precision with which the passages and chambers were constructed as the elements of a single coherent design. Although the objectives of the architect in devising such a design remain uncertain, the use of the inclined entrance passage as a geometrical structure is clearly shown, so that a significant insight is provided into the factors that determined the arrangement of the passages. In this article a similar analysis will be applied to the Second Pyramid of Giza - the Pyramid of Khaefre - the passage-system of which is almost as enigmatic as that of the Great Pyramid.

The formation of two entrance passages in Khaefre's Pyramid is usually explained on the assumption of a change of plan, although opinions differ markedly as to the nature of this change: either the pyramid was initially begun on a smaller base further to the north, so that the 'Lower Chamber', in its supposed role as the original burial chamber, would have been placed beneath the apex;² or else the pyramid had been planned on a larger scale, but was for some reason reduced in size by moving the north and east sides towards the centre.³ In neither case can evidence be found in the structure of the pyramid itself; but on the contrary, the fact that a core of rock was included in the pyramid to the height of four or five courses for the entire length of the west side, and to a lesser height for much of the east side, shows that the site has not been levelled to receive a smaller base; while a reduction in the size of the pyramid would not have required the construction of a second entrance passage, when the eventual burial apartment, the 'Great Chamber', was to be made accessible from the original entrance through a connecting passage.

We must therefore consider whether the passage-system in the Second Pyramid, which in any case displays a certain continuity of form, may actually represent architect's original intention. This possibility is strongly supported by the dimensional evidence, which suggests that the position of the Lower Chamber was carefully

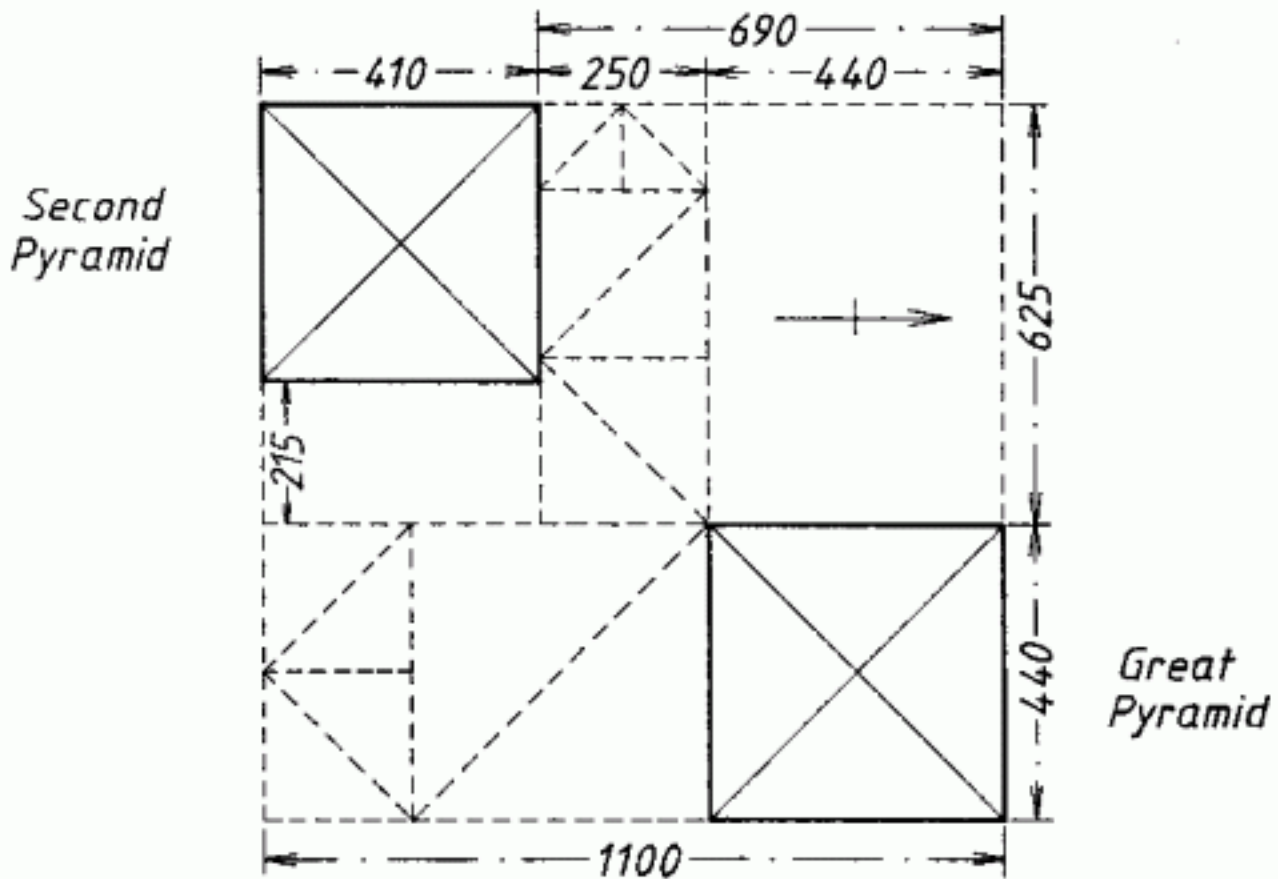


Fig. 1 Initial Scheme for Second Pyramid. Dimensions in Cubits.

calculated not only in relation to the Great Chamber, and to the dimensions of the pyramid itself, but also with reference to the position of the pyramid on the Giza plateau.

The Dimensional Scheme

As described by the writer elsewhere,⁴ the size and position of the Second Pyramid can be ascribed to a simple initial scheme in which the base extends from 250 to 660 cubits southwards from the south side of the Great Pyramid (see figure 1). The two pyramids thus span a north-south distance of 1100 cubits, or $5/2$ times the base of the Great Pyramid of 440 cubits, the base of the Second Pyramid being defined as $(660 - 250)$ or 410 cubits. The west side of the Second Pyramid is located $5/2 \times 250$ or 625 cubits westwards from west side of the Great Pyramid, so that the east-west spacing is defined as $(625 - 410)$ equals 215 cubits.

In the actual dimensions, however, an adjustment of one cubit is shown in the base of the Second Pyramid, which measures 3×137 or 411 cubits as shown by Petrie's survey result⁵ of 8474.9 inches equals 411.0 cubits, for the cubit of 20.620 inches or 0.52375 ms. The measured casing-angle of $53^{\circ} 10'$ is close to the theoretical

angle of $53^{\circ} 7' 49''$ for a profile of 4 rise on 3 base, so that the height was just 2×137 equals 274 cubits.

In Table I are listed the writer's own measurements of the lengths and angles of the passages, which are supported by the findings of Maragioglio and Rinaldi.⁶ Clinometer readings were taken over the entire length of each sloping passage and showed a very careful alignment for the two entrance passages; but the connecting passage was less accurately worked, as if aligned by stretching a cord along each part. The upper part was initially cut too far to the north, and the false start was filled up with masonry. From the computed horizontal and vertical lengths of the passages, the resulting positions inside the Second Pyramid are given in Table II, in metres and in cubits of 0.52375 metres. The original lengths of the two entrance passages have been found by reconstructing the missing casing and pavement.⁷

With reference to figure 2, it will be noted that instead of being positioned exactly in the mid-plane like the Queen's Chamber in the Great Pyramid, the Great Chamber in the Second Pyramid is situated entirely to the north of the central axis - the apex of its roof being about 197.10 cubits southwards from the north base. In this and in other dimensions, however, we will find that the architect developed the elements of his design not independently, but with allusion to the position of the Second Pyramid relative to the Great Pyramid; and that he worked not only in cubits but in palms (= 1/7 cubit), and digits (= 1/4 palm).

It thus appears that since the Second Pyramid was to be placed $(440 + 250)$ equals 690 cubits southwards from the north side of the Great Pyramid, the central axis of the Great Chamber (I) was set 2×690 equals 1380 palms, or 197.142 cubits, southwards from the north side of the Second Pyramid. The same principle is reflected in the vertical levels of the pyramid; and it will be seen from Table II that the Upper Entrance (L) is at a level of 24.64 cubits equals 690 digits above the base, which is also the level of the Lower Passage (BE) below the base. Four squares each measuring 2×690 digits can thus be constructed to mark off the horizontal distance of 2×690 palms southwards from the north base to the axis of the Great Chamber, while at the same time producing the levels of the Upper Entrance and Lower Passage, which are exactly

Table II **Computed Positions Relative to North Base**

Posn.	Distance South from Base			Level Below Base		
	Metres	Cubits	Design	Metres	Cubits	Design
A	- 9.33	-17.81	-17.857	0.0	0.0	0.0
B	23.14	44.18	44.142	12.90	24.63	24.642
C	31.02	59.23	59.142	"	"	"
D	"	"	"	15.34	29.29	29.285
E	38.90	74.27	74.142	12.90	24.63	24.642
F	52.39	100.03	100.000	7.18	13.71	13.700
G	61.37	117.17	117.142	3.55	6.78	6.785
H	100.74	192.34	192.392	"	"	"
I	103.23	197.10	197.142	"	"	"
J	45.35	86.59	86.571	"	"	"
K	42.77	81.66	81.620	"	"	"
L	9.69	18.50	18.482	-12.92	-24.67	-24.642

the horizontal length of the Connecting Passage, of 215/5 or 43 cubits. This length is divided at (F) in the ratio of 2:3, such that the vertical 'cut' at (F) is almost exactly 100 cubits southwards from the north base.

The Lower Chamber obtains its own level from a sloping passage, which descends 4.65 cubits below the Lower Passage (BE) to a level 29.3 cubits below the base. Again the use of palms and digits is indicated since this dimension is 820 digits, which is as many digits as there are cubits in the round-figure semi-perimeter of the Second Pyramid of 820 cubits. It is also one-quarter of the distance from the north base southwards to the junction between the lower and upper passage-systems (G), which is 117.17 cubits equals 820 palms; and hence the Lower Chamber together with the Connecting Passage will be encompassed by four squares of side 820 digits southwards from the north base, in a manner analogous to the placing of the Great Chamber. A dimension of 82 cubits is predominant in the Great Pyramid where it gives the level of the King's Chamber, which derives from a square based on the pyramid's height of 280 cubits.¹ In the Second Pyramid, the level of the Lower Chamber can similarly be obtained in a square of 100 cubits, as the remainder given when the side is marked off on the diagonal. The parts will be $100 \div \sqrt{2}$ equals 70.71 cubits, and $(100 - 70.71)$

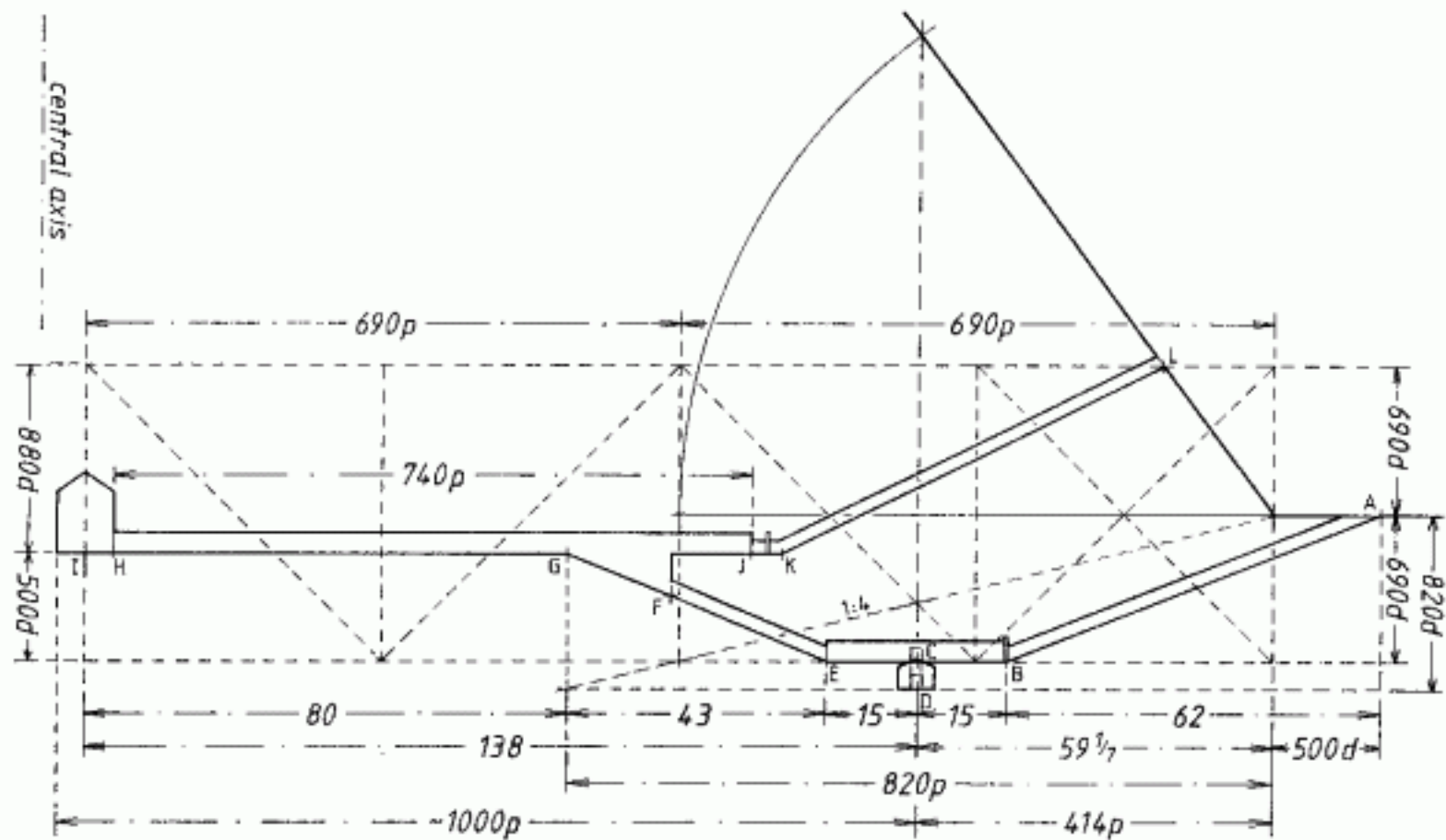


Fig.2 Dimensions of Second Pyramid in Cubits, Palms (p) and Digits (d).

equals 29.28 cubits, 205 palms, or 820 digits.

The careful placing of the Lower Chamber is further indicated by the fact that its east-west axis (CD) marks the exact mid-point of the Lower Passage (BE), since it is just 15 cubits from either end. The computed horizontal distance from this axis to the axis of the Great Chamber is 72.21 ms equals 137.9 cubits - or nearly $690/5$ equals 138 cubits, which is $7/10$ of the complete distance of 1380 palms from the pyramid's north base. The element from the north base to the axis of the Lower Chamber is therefore defined as (1380 palms - 138 cubits), or $59 \frac{1}{7}$ cubits - a dimension that can be constructed as shown in figure 2 by marking off a length of 690 palms onto the slope of the pyramid-casing. Since the slope corresponds to the hypotenuse of a 3:4:5 triangle, the subtended horizontal will be $\frac{3}{5} \times 690$ equals 414 palms or $59 \frac{1}{7}$ cubits.

At the same time, the south wall of the Great Chamber being (1380 palms + $9.5/2$ cubits) or nearly 202 cubits equals 1414 palms southwards from the north base, it is also about (1414 - 414) or 1000 palms southwards from the axis of the Lower Chamber. This is the side of a square, the diagonal of which measures $1000 \times \sqrt{2}$ or 1414 palms, so that a geometrical construction based on the ratio $1:\sqrt{2}$ is possible as shown for the level of the Lower Chamber, and for the King's Chamber in the Great Pyramid. The use of the same proportion is also established in the Giza ground plan.^B

For the junction (J) between the granite-lined Upper Entrance Passage and the Upper Horizontal Passage, a further reference is evidently made to the overall dimension of 202 cubits, from the north base to the south wall of the Great Chamber. With allusion to the slope of the pyramid of 4 rise on 3 base, this distance is divided in the ratio of 3:4, the point (J) being placed $\frac{3}{7} \times 202$ equals 86.57 cubits southwards from the north base (see Table II). When the remaining length of $\frac{4}{7} \times 202$ cubits is made the diagonal of a square, the side of this square gives the distance of 81.62 cubits from the lower end (K) of the Upper Entrance Passage to the north base. The Upper Entrance (L) being $\frac{3}{4} \times 690$ digits or 18.48 cubits horizontally inside the base, the horizontal length of the Upper Entrance Passage will be 63.14 cubits or 442 palms; but as the vertical length is 220 palms, the slope is slightly less than one rise on two base, with a defined angle of $26^{\circ} 27' 40''$. Since

the sloping length is then 70.5 or nearly $50\sqrt{2}$ cubits, it may be that the granite masonry, from the first course at the north base to the point (J), was intended to span $50\sqrt{3}$ equals 86.60 cubits.

The Lower Entrance Passage, briefly, terminates at about 44.2 cubits inside the base and has a sloping length of $100 \times 2/3$ cubits; the Connecting Passage begins 74.2 cubits inside the base and has a lower slope of 28 cubits with an upper slope of $74/4$ or 18.5 cubits. A factor 74 is also shown in the length (HJ) of the Upper Horizontal Passage, which is 105.75 cubits or 740 palms (see fig. 2).

Conclusions

Considering that most speculation as to possible changes of plan in Khaefre's Pyramid assumes a varying position for the north side of the pyramid, it is interesting to note that the exact position of this side, $(250 + 440)$ or 690 cubits southwards from the north side of the Great Pyramid, provided the modulus for the internal geometry and determined not only the position of the Great Chamber relative to the north base, but also that of the Lower Chamber. The round-figure side-length of the pyramid itself, of $(660 - 250)$ or 410 cubits, was an equally significant factor. The carefully-integrated positions of the two chambers suggests that both were conceived as elements of a single design - although the purpose of the Lower Chamber except as an expression of this design, still remains a matter of conjecture.

Notes

- ¹ Legon, J.A.R., Discussions in Egyptology 12 (1988), 41-48; Göttinger Miszellen 108 (1989), 57-64.
- ² Hölscher, U. Das Grabdenkmal des König Chephren (1912), 31-33.
- ³ Maragioglio, V. and Rinaldi, C.A., L'Architettura delle Piramidi Memphite Vol. IV (1966), 116-118.
- ⁴ Legon, J.A.R., Discussions in Egyptology 10 (1988), 33-40.
- ⁵ Petrie, W.M.F., The Pyramids and Temples of Gizeh (1883), 97.
- ⁶ Maragioglio and Rinaldi, op. cit. Tav. 6-10.
- ⁷ Existing length of Lower Entrance Passage, 34.01 ms, to sill 0.345 ms below and 8.46 ms north from base; existing length of Upper Entrance Passage, 33.88 ms to sill 11.54 ms above base.
- ⁸ Legon, J.A.R., Discussions in Egyptology 14 (1989, forthcoming).