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A KAHUN MATHEMATICAL FRAGMENT

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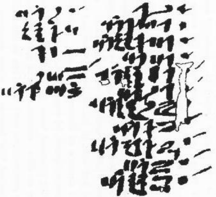
Among the papyri discovered by Petrie in the Middle Kingdom pyramid-town at El-Lahun were found some fragments dealing with mathematical problems, some of which were understood by Griffith,[1] while others were explained by Schack-Schackenburg.[2] The problem represented by columns 11 and 12 of Kahun fragment IV. 3, however, has not been fully understood, and much confusion has resulted from an analysis by R.J. Gillings.[3] In this article we will show that, contrary to Gillings' view, the text contains a straightforward and complete example of the Egyptian calculation of an arithmetical progression.

The significance of the numbers in column 12 of the fragment (see fig. 1) was in fact first recognised by Moritz Cantor,[4] who noticed that these numbers form the ten terms of an arithmetical progression with a common difference between the terms of 2/3 + 1/6 (or "3 + '6 to use the notation of fig. 1). Cantor also realised that since the sum of the ten terms is just 100, the hieratic signs for 100 and 10 which stand at the head of column 12, probably denote this sum and the number of terms, and not the number 110 which was transcribed by Griffith. If the scribe had intended to write the number 110, then the hieratic sign for 10 would be expected to stand above the tail of the 100 sign, so that the two signs could be read together as a single value; but the tail of the 100 sign in fact only runs into the side of 10 sign because of the cramped working, and the reading of these signs as two numbers is quite possible. The scribe has thus given a brief statement of the problem, which is to divide a quantity of 100 into 10 shares in arithmetical progression.

Evidently unaware of Cantor's article, however, Gillings took Griffith's transcription at face value, and tried at some length to account for the presumed total of 110 when the terms of the series only added up to 100. Because, as he pointed out, few examples of arithmetical progressions have survived in the mathematical papyri, Gillings subjected this problem to close scrutiny, and published two distinct solutions. The text was initially supposed to involve an arithmetical progression with seventeen terms, the smallest of which was equal to half the common difference while the sum of the

Col. 11 Col. 12



Col. 11	Col. 12
\ 1 '3 '12	100 10
2 "3 '6	13 "3 '12
4 1 "3	12 "3 '6 '12
\ 8 3 '3	12 '12
dmd 3 "3 '12	11 '6 '12
	10 '3 '12
	9 '3 '6 '12
	8 "3 '12
	7 "3 '6 '12
	7 '12
	6 '6 '12

Fig. 1 Kahun IV. 3

twelve largest terms was 110; but it was later suggested that the scribe had set out only to construct a progression in twelve terms adding to 110 with a common difference of "3 '6.

To explain why only ten terms of this series were actually given, when there was enough space on the papyrus for the scribe to write down several more terms if he had wanted to, Gillings wrote:[5] "we may surmise that he was checking his progression totals, and when he reached 100, he thought he had finished at 110. Or he may just have got tired of the interminable subtractions." In support of this theory, Gillings maintained that the working in the adjacent column 11 of the text was a check multiplication for the thirteenth term of the series, and a stumbling block to the view that the series was intended to contain only ten terms. In actual fact, however, this working is exactly that which is required by the Egyptian method of computing an arithmetical progression when ten terms are requested - as already proven by the other surviving example of a calculation of this type, in problem 64 in the Rhind Papyrus.[6] Since Cantor also failed to explain the significance of column 11, however, we will now describe how the calculation was carried out.

Given that a quantity of 100 has to be divided into ten shares in arithmetical progression, these shares will have nine differences between them, and the interval between the smallest and largest of the shares will equal nine times the common difference. The largest share can thus be found simply by adding half the total interval to the average share. This calculation was performed by the scribe by multiplying half the common difference by the number of differences; and hence in column 11 of Kahun IV.3, for the common difference of "3 '6, the scribe multiplies '3 '12 by 9 with a result of 3 "3 '12. This is added to the average share which is simply 100/10 equals 10; and hence as shown at the top of column 12, the largest share will be (10 + 3 "3 '12) equals 13 "3 '12. From this largest share, the common difference of "3 '6 is repeatedly subtracted to give each of the lesser shares in turn, down to the smallest share of 6 '6 '12.

Now to explain why the common difference of "3 '6 was selected, it will be noticed that the smallest share is about equal to half the largest share; and it seems very likely that an approximation to this relationship was the scribe's main objective. The problem was thus to distribute a quantity of 100 into 10 shares in arithmetical progression, such that the smallest share should closely equal half the largest share. The scribe may have surmised that the smallest and largest shares then had to represent one-third and two-thirds of their sum, which would equal twice the average share, or just 20; and that the correct values for these shares was therefore 6 "3 and 13 '3. But in this case, the common difference between the shares had to equal one-ninth of 6 "3 or "3 '18 '54, which was an awkward quantity to deal with. The calculation was made easier by rounding up the common difference to "3 '6, with slight error so far as any practical distribution was concerned.

In problem 64 in the Rhind Papyrus, on the other hand, it was required to divide 10 hekat of barley between ten men with a common difference equal to the Horus-eye fraction of '8 hekat. The largest share which resulted was thus perhaps quite arbitrarily rather more than three times the smallest share. Problem 40 in the Rhind Papyrus deals with the distribution of loaves in an arithmetical progression such that the two smallest shares amount to 1/7 of the three largest shares - a requirement which was clearly invented to make use of the chance property of a previously constructed progression. The Kahun fragment provides the only existing example where a distribution of shares in arithmetical progression appears to have been determined by a specific relationship between the smallest and largest shares.

NOTES

1. F.Ll. Griffith, Hieratic Papyri from Kahun and Gurob, 2 vols. (London, 1897). Vol.1, 16; vol.2, pl.VIII.
2. H. Schack-Schackenburg, Z.A.S. 37 (1899), 78-9; Z.A.S. 38 (1900), 138-9.
3. R.J. Gillings, Mathematics in the Time of the Pharaohs, (Cambridge, Mass., 1972), 176-180.
4. M. Cantor, 'Die mathematischen Papyrusfragmente von Kahun', Orientalistische Litteratur-Zeitung vol.1 no.10 (1898), 306-8.
5. Gillings op. cit., 180.
6. T.E. Peet, The Rhind Mathematical Papyrus (Liverpool, 1923), 107-8. See also G. Robins and C.C.D. Shute, The Rhind Mathematical Papyrus (London, 1987), 42-3.