

The Plan of the Giza Pyramids

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*My initial findings on the Giza Site Plan of Three Pyramids were first briefly summarized in a pamphlet published by the Archaeology Society of Staten Island in 1979.¹ Subsequently, I described the most significant features of the integrated site plan in several articles in the journals *Discussions in Egyptology*² and *Göttinger Miszellen*.³ Further research has shown that many more factors must be taken into account, without altering the basic framework of the dimensions in royal cubits which I described in 1979. The following text is based on my article in *Discussions in Egyptology Vol. 10*, but now includes much new material and new illustrations. In addition, account has been taken of the extensive survey work carried out since 2012 by the late Glen Dash, and the positions of the corners of the three pyramids have been placed in the coordinate system initiated by Mark Lehner and David Goodman for the Giza Plateau Mapping Project.*

Now that a detailed topographical study of the Giza Plateau is in progress,⁴ it is interesting to consider the results of the excavations and survey carried out by Flinders Petrie in 1880-2, when the exact dimensions and relative positions of the pyramids of Khufu, Khaefre and Menkaure, were established by triangulation..⁵ With reference to Petrie's survey-data, the present paper reviews the evidence first put forward by the writer in 1979,⁶ showing that the sizes and relative positions of the three pyramids were determined by a single unifying ground plan.

¹ J.A.R. Legon, 'The Plan of the Giza Pyramids', *Archaeological Reports of the Archaeology Society of Staten Island*, Vol.10 No.1. New York, 1979.

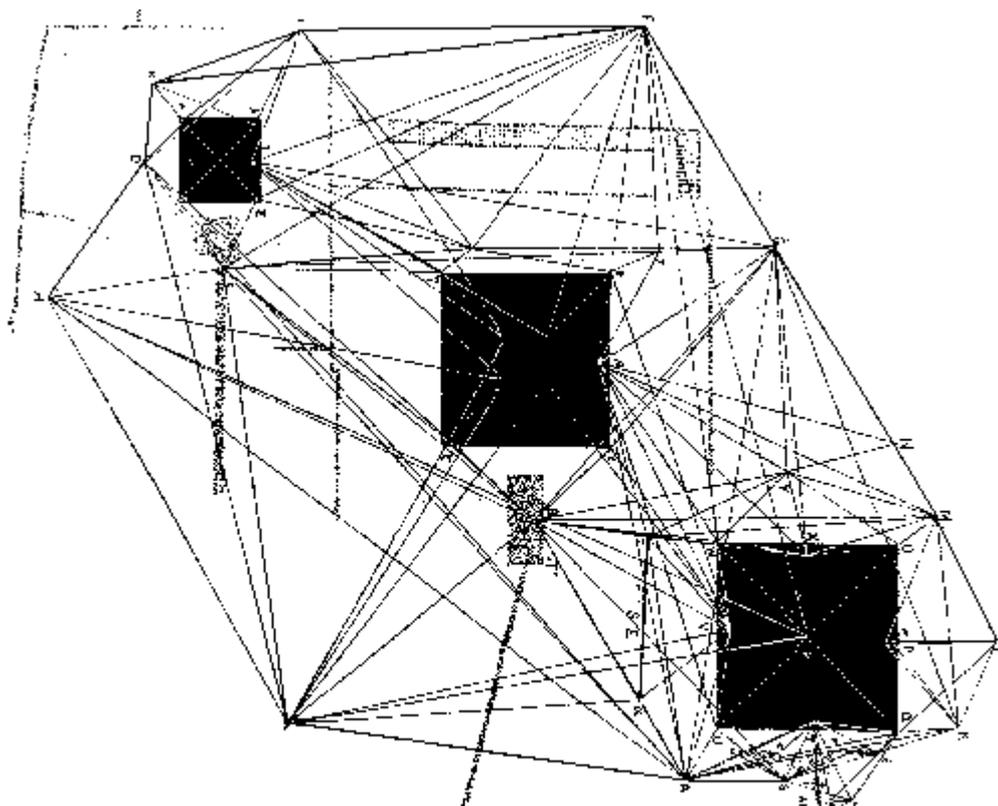
² J.A.R. Legon, 'A Ground Plan at Giza', *DE* 10 (1988), 33-40; 'The Giza Ground Plan and Sphinx', *DE* 14 (1989), 53-60.

³ J.A.R. Legon, 'The Design of the Pyramid of Khaefre', *GM* 110 (1989), 27-34; 'The Geometry of the Bent Pyramid', *GM* 116 (1990) 65-72, 71; 'The Giza Site Plan Revisited', *GM* 124 (1991), 69-78.

⁴ 1. M. Lehner, 'The Development of the Giza Necropolis: The Khufu Project', *MDAIK* 41, 1985, 109-143.

⁵ 2. W.M.F. Petrie, *The Pyramids and Temples of Gizeh* (London, 1883). First edition only for full details, 34-36.

⁶ J.A.R. Legon, 'The Plan of the Giza Pyramids', *Archaeological Reports of the Archaeology Society of Staten Island*, Vol.10 No.1. New York, 1979.



PLAN OF THE TRIANGULATION

OF THE GIZA PYRAMIDS CARRIED OUT BY W.M. FLINDERS PETRIE IN 1881-2

The existence of a dimensional scheme underlying the placing of the three pyramids is suggested in the first instance by the very regular arrangement of these pyramids on the Giza plateau. As a result, the sides of the bases and the distances that separate them define consecutive axial distances from north to south and from east to west. The three pyramids were accurately aligned with respect to the four cardinal points, and were displaced from one another in a configuration which satisfies the requirements of a coherent dimensional design. Certain technical difficulties relating to the site chosen for each pyramid in turn also suggest that there must have been some significant constraint, in addition to factors such as ease of construction or the selection of the most favourable architectural setting, which determined where each of the three pyramids was positioned.

Using some of the finest surveying equipment available in his day, Petrie asserted that he had fixed the positions of the main stations in his triangulation to within 3 mm.⁷ The accuracy of his work is proven by the fact that his result for the mean side-length of the Great Pyramid differs from the value obtained in the meticulous survey carried out by J.H. Cole in 1925,⁸ by only 1.5 cm, even though nearly all of the outer casing of the pyramid is missing. Whereas Petrie had located single points of the casing in pits near the centre of each side, excavations in preparation for the work by Cole showed that substantial traces of the original casing-edge still remained on the pavement in some places where the casing itself has been destroyed.

⁷ Petrie, *op.cit.*, 24.

⁸ J.H. Cole, *The determination of the exact size and orientation of the Great Pyramid of Giza, (Survey of Egypt, paper no.39)*, (Cairo, 1925).

The dimensions of the three pyramid-bases as determined by Petrie are given in Table I, together with the average variations in the lengths of the sides and the orientations of the three pyramids with respect to true north. The distances separating the centres of the pyramids were computed by Petrie along axes constructed parallel to the mean azimuth of the Second and Great Pyramids of $-4^{\circ} 52''$,⁹ and are given in Table II.

Table I	Petrie: Inches	Royal Cubits	Azimuth
Great Pyramid	9068.8 ± 0.6	439.81	$-3^{\circ} 43''$
Second Pyramid	8474.9 ± 1.5	411.00	$-5^{\circ} 26''$
Third Pyramid	4153.6 ± 3.0	201.44	$14^{\circ} 03''$

To obtain the axial components of spacing between the bases of the three pyramids, the distances between the pyramid-centres can be combined with the sides of the bases, to give the dimensions as listed in Table III. There will be small differences at the corners due to slight variations in the azimuths of the sides with respect to the axes of the plan; but since the Second and Great Pyramids have the same orientation to within two minutes of arc - a remarkably small divergence - these differences average only about 5 cm. The Third Pyramid, however, differs in azimuth from the Second and Great Pyramids by about $1/3$ degree in a clockwise direction, so that elements of about 25 cm are generated at the corners relative to the mean components of spacing. The exact positions can be calculated using Petrie's original survey coordinates (see below).

Table II: Axial Distances between the Centres of the Three Pyramids as stated by Petrie in Inches		
	N to S	E to W
Centre of 1st to Centre of 2nd Pyramid	13931.6	13165.8
Centre of 2nd to Centre of 3rd Pyramid	15170.4	9450.2
Centre of 1st to Centre of 3rd Pyramid	29102.0	22616.0

When the various dimensions are expressed in terms of the Royal Egyptian Cubit, with the value of 20.620 inches or 0.52375 metres as determined by Petrie from his measurements inside the Great Pyramid¹⁰ and as stated by Edwards,¹¹ it is found that almost all of the mean components of spacing correspond to whole numbers of cubits, or in some cases half numbers of cubits, to within 0.1 cubit. With reference to these potential design values, as listed in Table III, the largest error is only 0.23 cubit or 12 centimetres.

TABLE III: Axial Components of Spacing between the Sides of the Pyramids.			
Distances from North to South	Inches	Cubits	Design
N side 1st to S side 2nd Pyramid	22703.4	1101.04	1101
S side 2nd to S side 3rd Pyramid	13009.7	630.93	631

⁹ Petrie, op.cit.,125.

¹⁰ . Ibid, 179.

¹¹ I.E.S. Edwards, *The Pyramids of Egypt*, (Harmondsworth, 1947), 208.

N side 1st to S side 3rd Pyramid	35713.2	1731.97	1732
S side 1st to N side 2nd Pyramid	5159.7	250.23	250
S side 2nd to N side 3rd Pyramid	8856.1	429.49	429.5
Distances from East to West	...		
W side 1st to W side 2nd Pyramid	12868.8	624.09	624
W side 2nd to W side 3rd Pyramid	7289.5	353.52	353.5
E side 1st to W side 3rd Pyramid	29227.2	1417.42	1417.5
W side 1st to E side 2nd Pyramid	4393.9	213.09	213
W side 2nd to E side 3rd Pyramid	3135.9	152.08	152

The Lehner/Goodman and Glen Dash Surveys

In 1984, Mark Lehner and David Goodman began the Giza Plateau Mapping Project, which was intended to produce a topographical contour map of the plateau on a scale large enough to allow the positions of monuments and other archaeological features to be accurately recorded. In the following year, Lehner and Goodman published a map of the Giza plateau which some theorists assumed was the result of this new survey work. In fact, however, the various monuments and features were traced from the 1:5000 photogrammetric map which had been prepared by the Egyptian Ministry of Housing and Reconstruction, with the assistance of aerial photographs.

Although the EMHR map carries the framework of a grid which could have been used to locate structures on the plateau with sufficient accuracy for archaeological purposes, Goodman and Lehner decided to construct a new grid with an origin located at the notional centre of the Great Pyramid. This point does not exist on the ground and can only be deduced by reconstructing the pyramid's sides and corners. It was assigned the coordinates of 100,000 north and 50,000 east, in consequence of which the coordinates of structures west of the origin require the prefix of 499, while those to south of the origin are prefixed by 99 – a complication which could easily have been avoided by placing the origin outside the area of the plateau. Surprisingly, Lehner's grid is subject to an error of scaling of about 1 part in 50, so that the grid coordinates of the Third Pyramid, for example, are displaced by about 18 metres.

Confirmation of the accuracy of Petrie's findings has come from the surveys carried out in 2012 and 2015 by the Glen Dash Foundation in conjunction with Ancient Egypt Research Associates, continuing the work of Lehner and Goodman for the GPMP some thirty years earlier.¹² By locating Petrie's original station marks at the corners of the Great Pyramid, and referring to Petrie's stated coordinates for these stations, it has emerged that the standard of length implied by the base-line of Petrie's triangulation was effectively lacking by about 1 part in 5,000 – a discrepancy which Petrie himself noted when he compared his findings with the baseline of the previous survey by David Gill.¹³ Taking this into account in a significant endorsement of Petrie's work, Glen Dash

¹² Glen Dash, 'New Angles on the Great Pyramid', *Aeragram* 13-2, Fall 2012, 10-19

¹³ See Petrie, *Pyramids and Temples* p. 205-7. The shortfall may perhaps be ascribed to Petrie's base-line measuring equipment, which was of his own devising.

has converted Petrie's coordinates for the corners and centres of the pyramids into coordinates in the GPMP grid,¹⁴ to give the best possible estimates for these positions.

Petrie's coordinates for the corners of the Second and Third Pyramids, as transformed by Glen Dash to the cardinal orientation of the GPMP grid and expressed as northings and eastings, are listed in Table IV. The positions of the corners are then stated as southings and westings relative to the centre of the Great Pyramid. Next, the coordinates are transformed to the azimuth of the site plan of $-4^{\circ} 56''$ relative to the cardinal points and stated as distances southwards and westwards from the north-east corner of the Great Pyramid. Finally, these distances are converted into royal cubits of 0.52382 m or 20.623 inches.¹⁵ The average difference in the placing of the pyramid corners with respect to the proposed design values as shown is only 0.07 cubit or 3.6 cm.

Table IV		Petrie/GDF Coordinates of corners of Pyramids in GPMP grid aligned to cardinal points.	Distances southwards and westwards from centre of Great Pyramid at azimuth of GPMP grid		Distances southwards and westwards from N.E. corner of Great Pyramid at azimuth of site plan of $-4^{\circ} 56''$	Resulting Dimensions of Giza Site Plan in Royal Cubits of 0.52382 m or 20.623 in.		
Corner Location								
Second Pyramid		Metres		Metres	Metres	Cubits	Design	Diff.
N.E.	Northing	99753.437	Southing	246.563	361.420	689.969	690.0	0.03
	Easting	499773.458	Westing	226.542	342.078	653.044	653.0	0.04
S.E.	Northing	99538.127	Southing	461.873	576.730	1101.008	1101.0	0.01
	Easting	499773.857	Westing	226.143	341.987	652.872	653.0	0.13
S.W.	Northing	99537.773	Southing	462.227	576.775	1101.094	1101.0	0.09
	Easting	499558.503	Westing	441.497	557.342	1063.995	1064.0	0.01
N.W.	Northing	99753.092	Southing	246.908	361.456	690.038	690.0	0.04
	Easting	499558.231	Westng	441.769	557.305	1063.924	1064.0	0.08
Third Pyramid								
N.E.	Northing	99312.341	Southing	687.659	802.094	1531.239	1531.0	0.24
	Easting	499479.484	Westing	520.516	636.684	1215.463	1215.5	0.04
S.E.	Northing	99206.931	Southing	793.069	907.503	1732.471	1732.5	0.03
	Easting	499479.099	Westing	520.901	637.220	1216.487	1216.5	0.01
S.W.	Northing	99207.334	Southing	792.666	906.948	1731.412	1731.5	0.09
	Easting	499373.470	Westing	626.530	742.849	1418.137	1418.0	0.14
N.W.	Northing	99312.860	Southing	687.140	801.423	1529.959	1530.0	0.04
	Easting	499373.854	Westing	626.146	742.313	1417.115	1417.0	0.11

Analysis of the Giza Site Plan

To investigate the possibility of a deliberate positional relationship between the bases of the three pyramids, it seems reasonable to assume that any dimensional scheme would have been laid out starting from the base of the Great Pyramid, which was the first of the Giza pyramids to be

¹⁴ Glen Dash, 'Where, Precisely, are the Three Pyramids of Giza?'

http://www.academia.edu/6056783/Where_Precisely_are_the_Three_Pyramids_of_Giza

¹⁵ It will be noted that the azimuth and length of cubit differ very slightly from the values obtained in the writer's original analysis.

constructed. The dimensions would thus have been measured out from the north-eastern area of the plateau, southwards and westwards towards the Second and Third Pyramids. The task of setting out the plan would have been no more difficult in principle than laying out the base of a single pyramid, and it will be noted that the components of spacing between the three pyramids are in each case shorter than the side-length of the Great Pyramid.

Before the extensive quarrying of the rock for building stone took place, the natural surface of the Giza plateau had a general slope of around 5 degrees, and the projection onto the horizontal plane of a measurement taken over the surface would have been reduced by about 0.4%. Thus the dimensions of the site plan could have been laid out with reasonable accuracy without correcting for the slope of the plateau. Once the approximate position for each pyramid had been determined, however, the sites were leveled around the perimeter, and may also have been connected by temporary horizontal-topped embankments so that the east-west and north-south components of spacing between the three pyramid bases could be laid down with the greatest accuracy. While ropes may have been used in the initial marking out of the site plan, the observed accuracy in the finished dimensions was probably achieved through the use of pairs of butting measuring rods laid alternately end over end – a method capable of achieving a high degree of precision.¹⁶

The Great Pyramid

It is generally accepted that the sides of the Great Pyramid measure 440 cubits, although only the longest or south side has exactly this length according to the survey by Cole.¹⁷ In terms of the Giza royal cubit of 0.52375 metres or 20.620 inches, the actual mean side of 230.364 metres corresponds to 439.8 cubits, with an average variation in the sides of only 6 cm or 0.1 cubit. Petrie suggested that an adjustment had been effected such that the perimeter of the base would express the 'pi-proportion' in relation to the height of 280 cubits, with greater accuracy than the value for π of $22/7$.¹⁸ In this case, the theoretical side-length will be $(140 \times \pi)$ or 439.822... cubits.

Although the mean side-length of the Great Pyramid in the survey by the Glen Dash Foundation is just 1 mm less than Cole's mean result, small differences in the alignment of the individual sides give rise to variations in the side-lengths which cannot now be resolved. Cole's findings must take precedence over those of Glen Dash, however, because the traces of the casing-edge were better preserved. The use of regression analysis by the GDF team to determine confidence limits for the lengths of the sides is flawed because the data does not satisfy the statistical requirements.

The Placing of the Second Pyramid

Turning now to the position of the Second Pyramid relative to the Great Pyramid, the analysis of Petrie's survey data shows that the mean north-south spacing between the two pyramids is 250.2 cubits. It seems likely, therefore, that the builders intended to place the north side of the Second Pyramid on a line just 250 cubits southwards from the south side of

¹⁶ S. Clarke and R. Engelbach suggest an accuracy to one part in 250,000. *Ancient Egyptian Masonry* (Oxford 1930), 66.

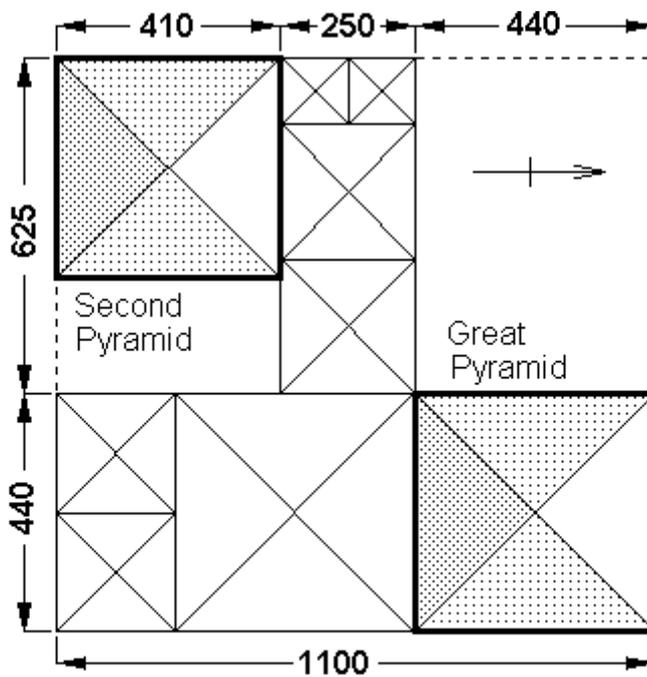
¹⁷ Length of south side, 230.454 metres; Cole, op.cit., 6

¹⁸ Petrie, op.cit., 220

the Great Pyramid. Taking further distances along the north-south axis, the impression of a deliberate design is strongly supported. The distance southwards from the north side of the Great Pyramid to the south side of the Second Pyramid is 1101 cubits, or only 0.1% greater than the round-figure of 1100 cubits. This is just $2\frac{1}{2}$ times the side of the Great Pyramid of 440 cubits, and the south sides of the two pyramids are separated by an axial distance from north to south of about $\frac{3}{2} \times 440$ or 660 cubits.

From these plausible formative dimensions of 440 and 250 cubits, we can now obtain a provisional design-value for the side-length of the Second Pyramid as follows:

Nominal Side-length of Second Pyramid = $440 \times \frac{3}{2} - 250 = 410$ cubits



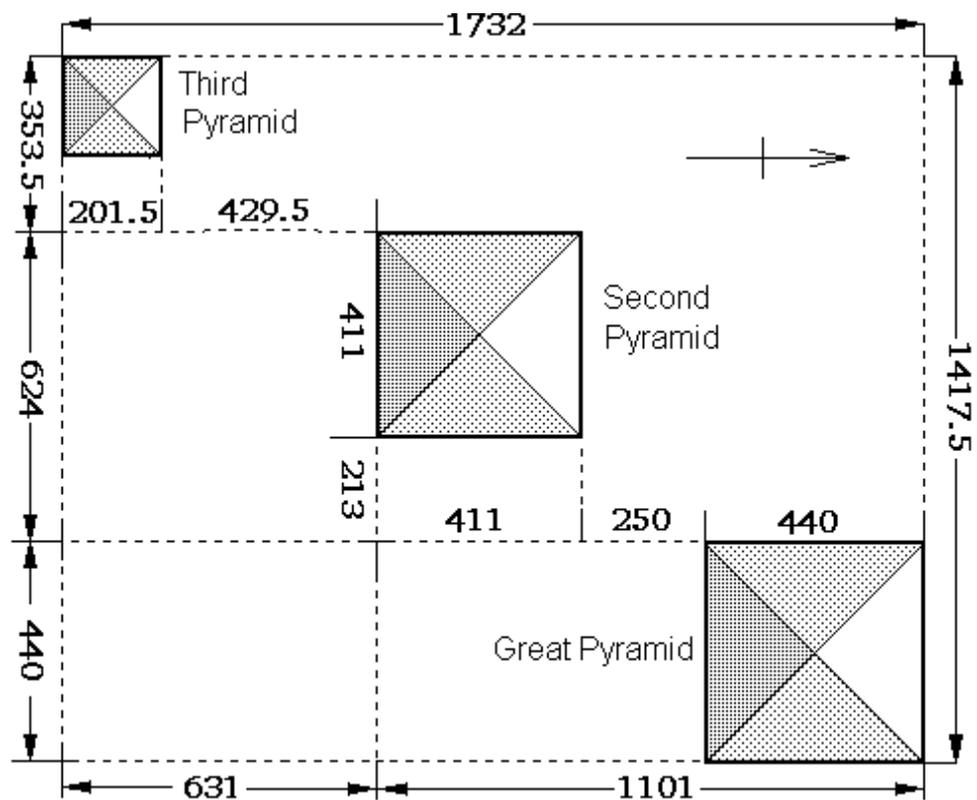
Modular Scheme Connecting the Second and Great Pyramids
Dimensions in Royal Egyptian Cubits

I have termed this the nominal side-length of the Second Pyramid, because Petrie's survey-data gives an actual mean side of 8474.9 inches or exactly 411 cubits. The average variation in the sides from their mean length is only 1.5 inches or 4 cm. Thus although the above derivation accounts for a dimension of about 410 cubits in preference to one of just 400 cubits, an adjustment of one cubit appears to have been made to the actual side-length. Reasons for this adjustment will be given shortly.

Now taking dimensions along the east-west axis, Petrie's data places the west side of the Second Pyramid 624 cubits westwards from the west side of the Great Pyramid. Again with an adjustment of one cubit, therefore, this dimension may be equated with a nominal value of $2\frac{1}{2} \times 250$ or 625 cubits, making it analogous to the distance of $2\frac{1}{2} \times 440$ or 1100 cubits along the north-south axis. Hence the side-length of the Second Pyramid and the position relative to the Great Pyramid may be ascribed to a simple design based upon modules of 440 and 250

cubits. The side-length of the Second Pyramid is defined as $(1\frac{1}{2} \times 440 - 250)$ or 410 cubits, while the east-west spacing from the Great Pyramid is found to be $(625 - 410)$ or 215 cubits:

With reference to this modular scheme, however, the builders evidently subtracted one cubit from the dimension of 625 cubits, and added one cubit to the dimension of 1100 cubits, making the side-length of the Second Pyramid equal to 411 cubits, while the east-west spacing from the Great Pyramid became $(624 - 411)$ or 213 cubits (see Table III). These adjustments suggest that further factors influenced the final choice of dimensions; and indeed these factors are found to have anticipated the inclusion of the Third Pyramid in the completed ground plan.



Dimensions of the Giza Site Plan in Royal Egyptian Cubits

The Enclosing Rectangle of the Site Plan

As shown by the survey-data in Table III, the Third Pyramid extends the scheme of the Second and Great Pyramids by 631 cubits towards the south and 353.5 cubits towards the west. The overall dimensions of the Giza site plan are thereby defined along the two axes. Computing these enclosing dimensions from the component parts, we find:

Overall East-West Dimension = $440 + 624 + 353.5 = 1417.5$ cubits

Overall North-South Dimension = $440 + 661 + 631 = 1732$ cubits

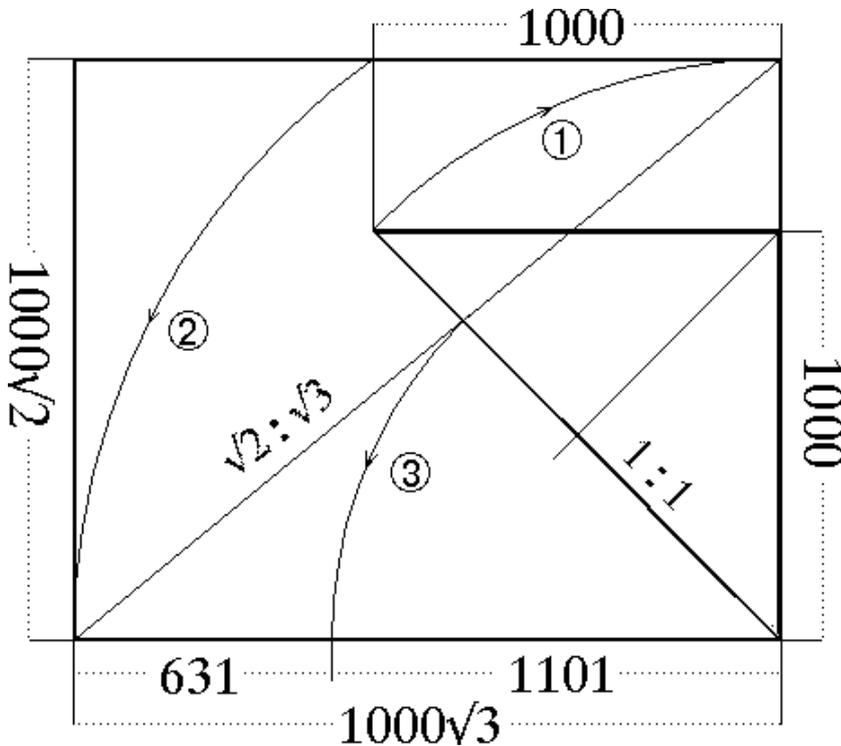
Rather surprisingly, these dimensions represent the square roots of the first two prime numbers, two and three, multiplied by a factor of 1000. The theoretical values are:

$$1000\sqrt{2} = 1414.21\dots, \quad 1000\sqrt{3} = 1732.05\dots$$

While the north-south dimension thus expresses the value of $1000\sqrt{3}$ exactly, the east-west dimension displays a discrepancy of about 3.3 cubits with respect to the exact value of $1000\sqrt{2}$. This apparent error amounts to only about 0.2%, however, and is explained by other factors in the formulation of the dimensions, as we will see.

The occurrence of these square root values in the dimensions of the Giza plan points to a geometrical development beginning with a square of 1000 cubits, as shown in the diagram below. The east-west dimension of $1000\sqrt{2}$ cubits is first obtained as the diagonal length in this square, which is marked off along the north side to give the overall dimension from east to west. A rectangle measuring 1000 by $1000\sqrt{2}$ cubits is then constructed, with a diagonal length of $1000\sqrt{3}$ cubits which is marked off along the east side to give the overall dimension of the plan from north to south.

The validity of this geometry is confirmed by the ease with which it can be extended so as to define the major division of the plan which takes place at the south side of the Second Pyramid. It is in fact only necessary to draw in the diagonal of the enclosing rectangle on a plan of $\sqrt{2} : \sqrt{3}$ to obtain a point of intersection with the opposing diagonal of the initial square on a plan of $1 : 1$. This defines a length of diagonal equal to the dimension of 1101 cubits which encloses the Second and Great Pyramids from north to south, as shown below .



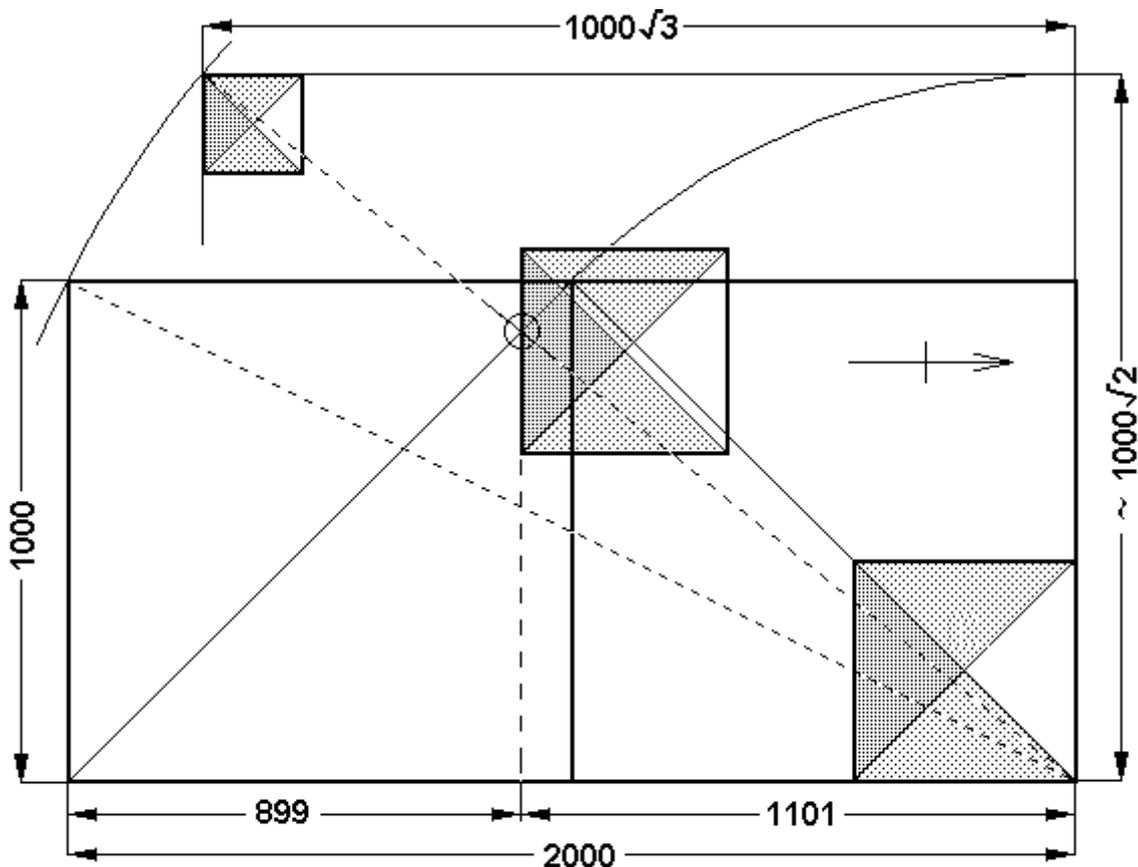
It will be seen that the diagonal rising from southern end of the base-line of $1000\sqrt{3}$ cubits on a plan of $\sqrt{2} : \sqrt{3}$, intersects the diagonal rising from northern end on a plan of $1 : 1$, such as to

divide the base-line in the ratio of $\sqrt{2} : \sqrt{3}$. Consequently, the length marked off along the 1 : 1 diagonal will be:

$$1000 \sqrt{3} \times \frac{\sqrt{2}}{\sqrt{2} + \sqrt{3}} \times \sqrt{2} = 1101.020\dots \text{cubits}$$

Hence nearly the exact whole number of 1101 cubits which we earlier ascribed to a simple modular scheme adjusted by one cubit, is now seen to have had its origin in the most fundamental geometry of the enclosing rectangle of the site plan.

It is also possible to construct the enclosing rectangle starting from a double square or 1:2 rectangle of 1000×2000 cubits. While the east-west dimension of $1000\sqrt{2}$ cubits is given once again by the diagonal in each component square of 1000 cubits, the north-south dimension is constructed with reference to the diagonal of the double square of $1000\sqrt{5}$ cubits. When this dimension is applied as the hypotenuse in a right triangle having the dimension of $1000\sqrt{2}$ cubits as base, the height of the triangle will be $1000\sqrt{3}$ cubits. The hypotenuse, furthermore, intersects the opposing diagonal of one component square so as to divide the length of the double square in the exact ratio of $\sqrt{2} : \sqrt{3}$, or into parts of 899 and 1101 cubits. Thus the major north-south division of the site plan is defined as before.



Overall Dimensions of the Site Plan developed from a 1000 x 2000 cubit rectangle showing division at the south side of the Second Pyramid in ratio $\sqrt{2} : \sqrt{3}$

Needless to say, these geometrical constructions are conceptual and would have been worked out by the architect on a convenient scale on a drawing board or scribing floor. Only the pyramid bases and the components of spacing between them had to be marked out on the ground of the Giza plateau.

Completion of the Site Plan

The evaluation of Petrie's survey-data shows that the north-south overall dimension of the Giza site plan together with the division at the south side of the Second Pyramid, satisfy the geometrical requirements with complete accuracy. The question therefore arises as to why the east-west overall dimension diverges from the value of $1000\sqrt{2}$ or 1414.2 cubits by about three cubits. That this is not an error is initially indicated by the fact that the axial component of 353.5 cubits westwards from the Second Pyramid to the west side of the Third Pyramid has the value of $250\sqrt{2}$ cubits:

$$250\sqrt{2} = 353.55\dots \text{ cubits}$$

Thus this dimension is accurately one-quarter of the ideal overall dimension on the same axis. It is the length of the diagonal in the modular layout squares of 250 cubits, which define the north-south spacing between the Second and Great Pyramids and the dimension of about $2\frac{1}{2} \times 250$ or 625 cubits on the east-west axis. Adding this element of 353.5 cubits to the distances as already defined in our initial modular scheme along the east-west axis gives an overall east-west dimension of $(440 + 625 + 353.5)$ equals 1418.5 cubits, which is more than 4 cubits greater than the value of $1000\sqrt{2}$ cubits. Thus the subtraction of one cubit from the component of $2\frac{1}{2} \times 250$ or 625 cubits may be seen as a judicious adjustment which improved the accuracy of the square-root value in the overall dimension, without unduly undermining the integrity of the modular scheme. This adjustment, furthermore, was balanced by the addition of one cubit to the modular dimension of $2\frac{1}{2} \times 440$ or 1100 cubits, to satisfy the fundamental division of the overall dimension at the south side of the Second Pyramid.

The mathematical import of the site plan is greatly enhanced by the expression of the square roots of 2, 3, 5 and 7, in each case multiplied by the module of 250 cubits. The side-length of the Second Pyramid being 411 cubits, the axial distance westwards from the centre of the Second Pyramid to west side of the Third Pyramid is:

$$411/2 + 353.5 = 559 \text{ cubits}$$

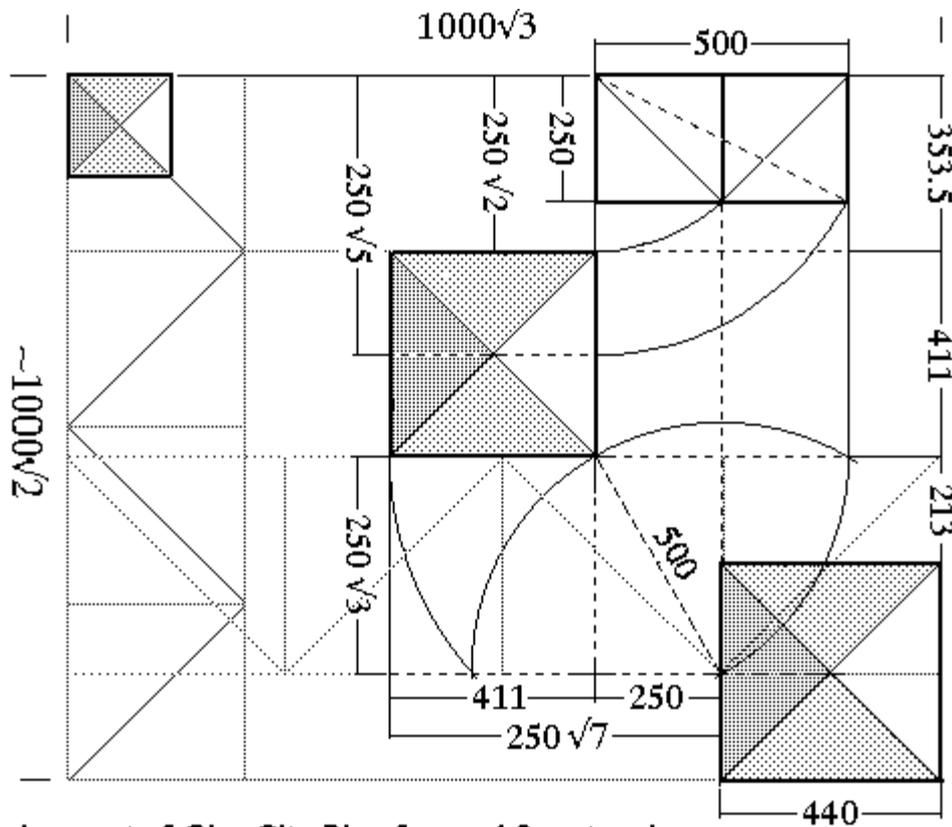
$$\text{and } 250\sqrt{5} = 559.01\dots$$

The east-spacing between the Second Pyramid and Great Pyramid being $(624 - 411)$ or 213 cubits, the axial distance westwards from the centre of the Great Pyramid to the east side of the Second Pyramid is found to be:

$$440/2 + 213 = 433 \text{ cubits}$$

$$\text{and } 250\sqrt{3} = 433.01\dots$$

This is the length of diagonal in the rectangle measuring $250 \times 250\sqrt{2}$ cubits, which is contained in the Giza site plan between the north-south dimension of 250 cubits and the east-west dimension of 353.5 cubits. It is now evident that a $1 : \sqrt{3} : 2$ right triangle with hypotenuse of 500 cubits and adjacent sides of 250 and 433 cubits can be constructed on the spacing between the Second and Great Pyramids, as shown below.



Development of Giza Site Plan from a 1:2 rectangle measuring 250 x 500 royal Egyptian cubits

Finally, the axial distance southwards from the Great Pyramid to the south side of the Second Pyramid is:

$$(250 + 411) = 661 \text{ cubits}$$

$$\text{and } 250\sqrt{7} = 661.437\dots$$

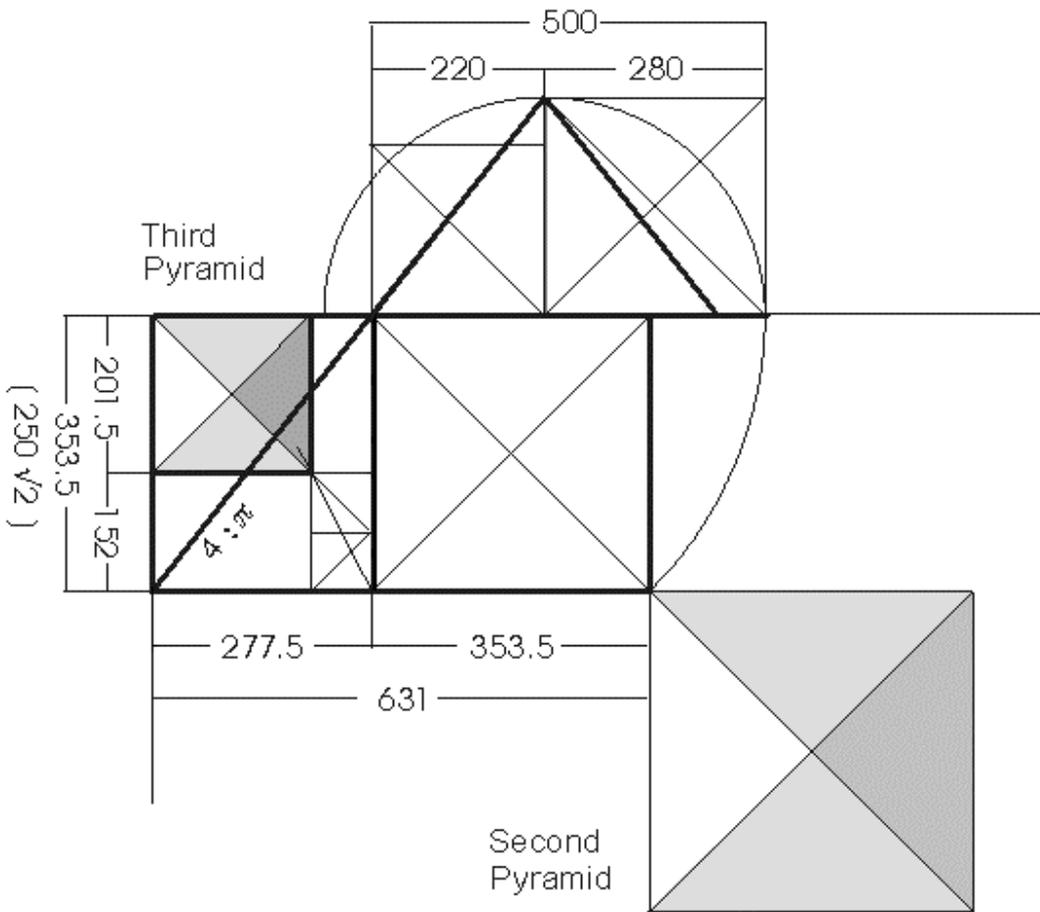
In closely connected dimensions along the two axes of the plan, therefore, the square roots of two, three, five, and seven are all accurately represented in terms of a module of 250 cubits. Three of these square-roots values are represented elsewhere in the plan with a factor of 1000 cubits. Given that the site plan contains only seven independent dimensions – the three pyramid bases and the four components of spacing - the fact that seven square root values are expressed in the dimensions is remarkable.

It will be seen that the addition of one cubit to the nominal side of the Second Pyramid of 410 cubits facilitated the accurate expression of $250\sqrt{3}$ and $250\sqrt{5}$ on the east-west axis, and also resulted in a good approximation to $250\sqrt{7}$ on the north-south axis. This last

dimension is a natural product of the geometry already described., and can be constructed as the diagonal in a rectangle measuring $250\sqrt{2}$ by $250\sqrt{5}$ cubits, or as the diagonal in a rectangle measuring $250\sqrt{3}$ by 500 cubits. In conclusion, the square roots of the first four prime numbers are all defined in the Giza plan in terms of the module of 250 cubits., which defines the north-south spacing between the Second and Great Pyramids.

The Scheme of the Third Pyramid

The Third Pyramid seems to have served primarily as a corner marker in the Giza site plan, and thereby extends the scheme of the Second and Great Pyramids by 353.5 cubits towards the west and 631 cubits towards the south to define the enclosing rectangle of the plan.. It has dimensions of its own, however, which represent a significant development of the design. It so happens that the numbers 353.5 and 631 are in the almost exact ratio of 1: $(1 + \pi/4)$, and can therefore be used to generate the pi-proportion of the Great Pyramid within the bounding 353.5 by 631 cubit rectangle

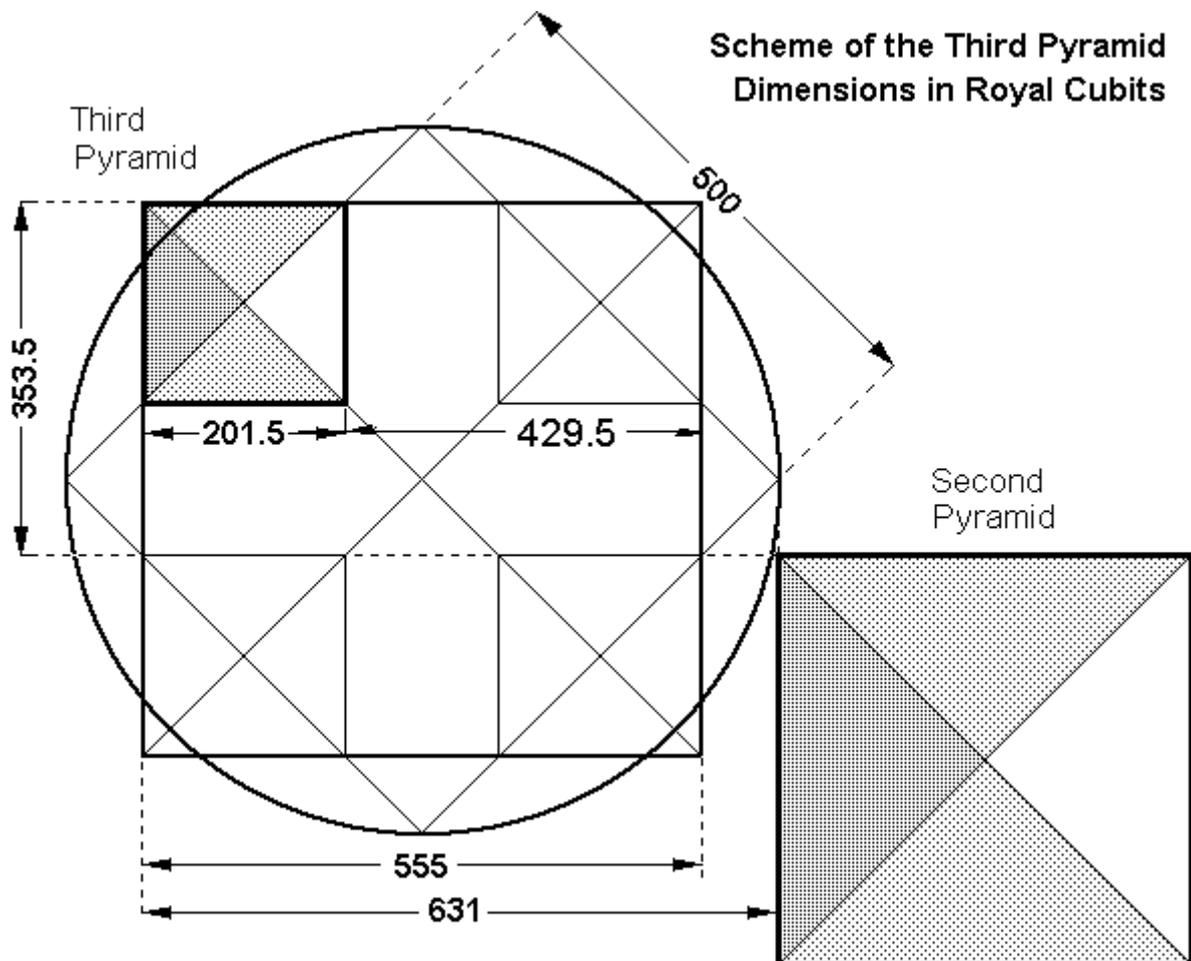


Development of the bounding rectangle of 353.5 x 631 cubits to give the base of the Third Pyramid and the pi-proportion. The height and base of the Great Pyramid are also shown.

Subtracting the short side from the long side leaves a remainder of $(631 - 353.5)$ or 277.5 cubits, which is in the ratio of $\pi : 4$ to the short side of 353.5 cubits and so yields the $\pi/4$ pyramid slope:.

As shown in the diagram above, the side-length of the Third Pyramid is defined within the rectangle of 277.5 by 353.5 cubits by the intersection of a line drawn on a plan of 1:1 from the SW corner, with a line drawn on a plan of 1:2 from the NE corner. This results in a space on the east side which is twice as wide as the space on the north side, giving dimensions of 152 and 152/2 cubits respectively. The side-length of the Third Pyramid is thus found to be $(353.5 - 152) = (277.5 - 76) = 201.5$ cubits, in agreement with Petrie's mean survey result of 4153.6 inches or 201.44 cubits.. The element of 152 cubits determines the east-west spacing between the Second and Third Pyramids, and is exactly one-seventh of the consecutive axial distance of 1064 cubits from the west side of the Second Pyramid to the east side of the Great Pyramid..

These relationships form the basis for a scheme of the circle squared, in which a square with a side of 500 cubits is circumscribed by a circle with radius of $250\sqrt{2}$ or 353.5 cubits, and a circumference of 2220 cubits (for $\pi = 3.140$). To square the circle, a second square with a side of 555 cubits is centred on the first square but aligned along the diagonal, generating the elements as described above.



We must also note that the base of the Third Pyramid is equal to the side of a cube, the volume of which is that of a sphere with a diameter equal to the module of 250 cubits. The exact side-length required for this cubing of the sphere being 201.499 cubits, the result is not lacking in accuracy.

The Orientation of the Third Pyramid

The occurrence of half numbers of cubits in the planning of the Third Pyramid is associated with the slight difference in the orientation of this pyramid with respect to the axes of the site plan, sufficient to shift the corners by 0.5 cubit in a clockwise direction relative to the midpoints of the sides. By this means, whole tens of cubits were introduced into dimensions which would otherwise have been greater or less by 0.5 cubit, and a secondary set of requirements for the dimensions relating to the Third Pyramid was facilitated.

The north-south spacing between the Second and Third Pyramids was thus increased to 430 cubits when measured to the north-east corner of the Third Pyramid, but reduced to 429 cubits when measured to the north-west corner. The lesser dimension combines with the side-length of the Second Pyramid to give a distance of $(429 + 411)$ or 840 cubits, which is just three times the height of the Great Pyramid. It will also be seen that the north-west corner of the Third Pyramid is $(429 + 1101)$ or 1530 cubits southwards from the northern boundary of the plan, while the south-west corner is $(354 + 411)$ equals 765 or $1530/2$ cubits westwards from the east side of the Second Pyramid. There is thus a hint of the further relationships which are hidden within structure of the plan.¹⁹

Conclusion

Whilst the wealth of evidence here presented leaves no doubt whatsoever that the three great Giza Pyramids are the components of a single unified site plan of surprising sophistication, it must be admitted that the existence of the plan is entirely unexpected by Egyptologists, and indeed makes little sense in the context of ancient Egyptian funerary architecture. The Egyptian kings were not given to forming positional relationships between their tombs and those of their predecessors and successors, and the mathematical nature of the design at Giza can hardly be explained by any known requirements of the funerary cult.

Whereas the Egyptologist will see this as a reason to dismiss the site plan as a strange aberration that can have no relevance for their subject, the true scientist will recognise the impossibility of the mathematical relationships having arisen by chance, especially given the accuracy, simplicity and coherence of the design, and the consistency with which a particular theme – the expression of square roots – was pursued by the architect. It is possible to envisage the occasional approximate occurrence of such numbers popping up in random sets of dimensions by chance, but the systematic and accurate expression of the square roots of the first four prime numbers with a factor of 250 cubits, duplicated in three instances with a factor of 1000 cubits, cannot possibly be dismissed as coincidence.

¹⁹ . These results point to a further geometrical development of the plan as described in 'The Giza Site Plan Revisited' in *Göttinger Miszellen* 124 (1991).

If, as the present writer believes, the existence of the site plan cannot be reconciled with the funerary theory, then it is not the solid factual evidence of the dimensional relationships that is at fault, for this is the bed-rock reality of the Giza site. The evidence for the usage of these pyramids as tombs by certain kings of the Fourth Dynasty, on the other hand, is secondary and superficial, and could be explained by the appropriation of pre-existing monuments. The Giza layout provides compelling evidence in support of the claim which is made in the Osirian Scripts,²⁰ to the effect that the three pyramids were constructed as a centre for the initiation of priest-neophytes into the occult wisdom of ancient Egypt, some thousands of years before the founding of the historical First Dynasty.

Needless to say, the notion that the Giza pyramids and temples could predate the Fourth Dynasty is unlikely to find many supporters in orthodox circles, but is nonetheless worthy of consideration in the view of the present writer.. The fact that the name of Khufu is found written on the internal masonry of the Great Pyramid in no way invalidates this possibility, since it is obvious that a king who wished to claim ownership of the monument would have been obliged to adopt the name of the original builder as his own, in order to avoid the need to erase the name from the fabric of the pyramid – an impossible task.

The historical Khufu of the Fourth Dynasty would not, however, have wanted to be remembered as the appropriator of the Great Pyramid, but rather as the builder, and to this end it is conceivable that he required the population of Egypt to contribute stones towards its construction. Garbled references to this state of affairs may be found in the history of Herodotus, who tells us that Khufu plunged into all manner of wickedness. He closed the temples, forbade the Egyptians from making offerings to the gods, and instead compelled them to labour one and all in his service.²¹ He raised money by prostituting his own daughter, who being bent on leaving a monument which would perpetuate her own memory, required each man to make her a present of a stone towards the works which she contemplated.. The records concerning the delivery of stones to the site which have recently been found at the port of Wadi el Jarf on the coast of the Red Sea have been assumed to support the conventional view of Khufu's activities, but could equally show how officials were required to deliver stones to the Giza site and keep a detailed account of their contribution to the king's faux pyramid-building exercise..

Leading Egyptologists have long recognised the difficulty of maintaining that a monument as vast as the Great Pyramid could have been built in the reign of one king.. The work load in terms of blocks of stone moved into position each day was simply too great. The idea that the Giza Pyramids predate the Fourth Dynasty may seem preposterous to many, but is justified by the records preserved by the Egyptians themselves in documents such as the Turin King List and the history of Manetho. Here we find mention of an era of remote antiquity known as *Zep Tepi* – the First Time - when Egypt was ruled by priest-initiates who founded the divine dynasties and were later revered as gods or demigods. Scattered across Egypt are monuments that date back to this era but are mostly deeply buried beneath the silt of the Nile, although some were restored and reused by the kings of the 'authentic' period..Amongst these monuments, its position carefully integrated with the Giza plan, the Great Sphinx still stands as the enduring guardian of the genuine mystery regarding which I have endeavoured throw a glimmering light.

²⁰ See H.C. Randall-Stevens, *Atlantis to the Latter Days*. Jersey, 1966. <http://www.cdfas.plus.com/kta.htm>

²¹ The History of Herodotus, Book II Chap.124-6.