

THE GIZA GROUND PLAN AND SPHINX

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In a previous article the writer has described the coherent ground plan in which the three large Pyramids of Giza are connected in a single unifying scheme [1]. The existence of this plan raises a number of difficult questions, not least of which is the problem of understanding why the pyramid-builders of the Fourth Dynasty should have embarked upon such an ambitious project, if indeed the plan had been envisaged at the outset. Our present purpose is to consider some of these questions, and to indicate the manner in which the plan was extended to include the pyramid-temples, the connecting causeways, and even the Great Sphinx.

Development or Grand Design?

It might be supposed that the simple relationship by which the Second Pyramid of Khaefre is connected with the Great Pyramid of Khufu, would have been devised after the Great Pyramid had been constructed, when pyramid-building was resumed at Giza following Djedefre's failure to complete his monument at Abu Rowash. It is hard to believe that Khufu's architects would have envisaged the construction of the Second Pyramid, and indeed also of the Third Pyramid of Menkaure, and have left space for their construction on the Giza plateau when circumstances would allow them to resume their 'grand design'. Yet it is precisely this conclusion, that the Giza ground plan was conceived in its entirety at the outset and was not merely the result of a gradual development, that appears to be proven both in the choice of the dimensions and in the site chosen for each pyramid.

The situation of the Great Pyramid, in particular, strongly suggests that Khufu's builders had anticipated the construction of further pyramids on the Giza plateau; for if the entire area of the plateau was at their disposal, the question arises as to why they chose not the highest ground - which offered both the most central position and the easiest means of approach - but instead selected a location on the lower part of the plateau, very close to the northern cliff. This site involved the construction of a

massive stone ramp to support the causeway in its ascent of the eastern cliff, to a height of more than 30 metres - a work said by Herodotus to be not much inferior to the pyramid itself.

The natural position for the Great Pyramid as an independent monument would surely have been near the crest of the plateau, north-westwards from the Second Pyramid, where Khufu could rest assured that the grandeur of his tomb would remain unassailable. A causeway leading up a natural incline from about the position of the Sphinx could then have been built, saving considerable labour and placing the valley temple in an ideal position at the foot of the plateau. Instead, however, by locating the Great Pyramid on the lower part of the plateau, the builders left the high ground for Khaefre, with the result that despite its lesser dimensions, the Second Pyramid effectively dominates the scene.

In the dimensional scheme connecting the Great Pyramid to the Second Pyramid, the latter was placed on an area of sloping rock, the levelling of which involved cutting an escarpment reaching 12 metres in height along the north and west sides, and building a megalithic foundation platform around the south-east corner. This scheme was not, however, final, since adjustments of one cubit relating only to the inclusion of the Third Pyramid in the complete design, were made in the defined distances along both axes of the plan [2]. The position of the Third Pyramid can be ascribed in part to a geometrical development based on squares of 1000 cubits, creating a large rectangle which determined the overall dimensions of the ground plan, and hence the limiting positions of both the Third and Great Pyramids on the plateau.

These factors lead us to conclude that the three pyramids were conceived in a single 'grand design', since a step by step exploitation of the plateau from its virgin state would probably have led to a different outcome. Can Khufu himself be regarded as the initiator of this plan, or were there other forces at work that influenced the course of events? No certain answer can yet be given, although the laying out of cemeteries around the Great Pyramid in a manner almost without parallel at other sites, in regular rows and columns of similar core-mastabas, was clearly a facet of this same idea, which sought to control all building activities at Giza for an extended period of time.

The Causeways and Sphinx

As previously indicated, the laying out of the Giza ground plan was not limited to the three pyramids themselves, but included the pyramid-temples, the causeways, and also the Great Sphinx. Reference to a plan such as that published by Reisner [3] will show that while the causeway of the Third Pyramid is aligned due east-west, the causeways of the Second and Great Pyramids both have a bearing of 14° - the former to the south and the latter to the north of due east. As noted by Lauer [4], this angle gives an offset of just one part in four, thus establishing the dimensional relationship between the ends of the causeways.

While the causeway of the Great Pyramid terminates at present beneath the village of Nazlet es-Simman, the temple-complex of the Second Pyramid is of course well known, and has been mapped to a scale of 1:600 in a large plan published by Selim Hassan [5]. This plan extends from the base of the Second Pyramid to the Lower Temple and Sphinx and carries a grid of 25-metre squares, making it possible to place the Sphinx in a coordinate system from which its place in the ground plan can be determined. Further details can be obtained from Hölscher's 1:150 plans of the Upper and Lower Temples [6], and Ricke's 1:150 plan of the Sphinx-temple [7].

In Table I, the coordinates of the key points in this layout are given as measured off from the grid-origin in Hassan's survey map, and then computed with respect to the north-east corner of the Great Pyramid. To determine the relative positions, the axis of the upper end of the causeway of the Second Pyramid is placed in the plan by Hölscher 9.7 ms south and 120.6 ms east from the Second Pyramid at the centre of its east side - this latter being 469.04 ms south and 341.95 ms west from the north-east corner of the Great Pyramid from Petrie's survey [8]. The ground plan is estimated to have an azimuth of $-35'$ relative to the grid of the survey map, the coordinates being transformed to this azimuth.

Relative to the axes of the plan, the causeway of the Second Pyramid is now found to measure 119.7 ms from north to south by 477.5 ms from east to west - the latter being almost equal to the north-south distance from the west end of the causeway to the north base of the Great Pyramid. From Table I this distance is 478.74 ms or 914.1 cubits, and may be taken as 914.28 cubits or

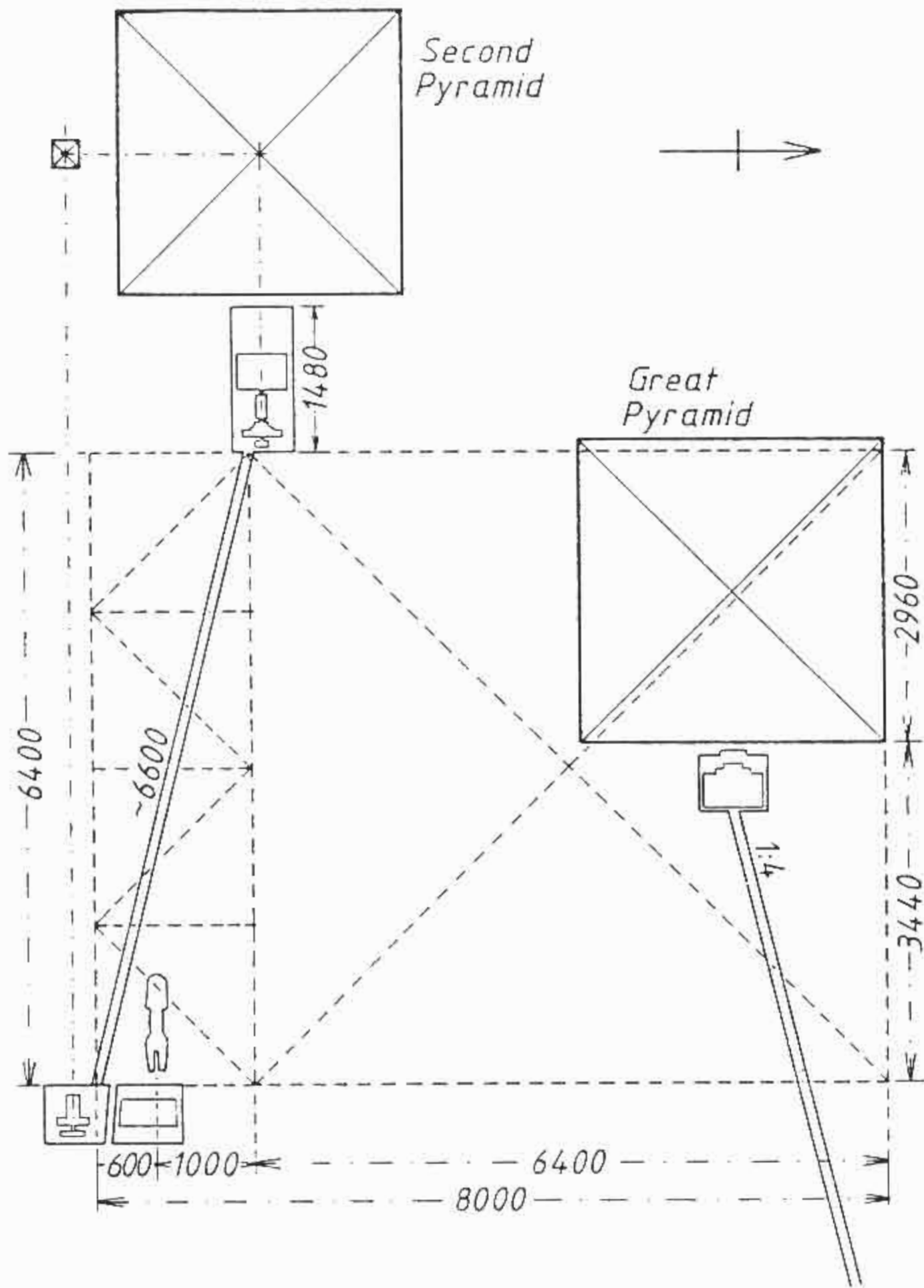


Figure 1. Dimensions of the Causeway and Sphinx in Palms.

The placing of the Sphinx, 7400 palms southwards from the north base of the Great Pyramid, is reflected in the length of the Upper Temple of $\frac{1}{5} \times 7400$ palms, and position of the causeway threshold $\frac{2}{5} \times 7400$ palms from the east base of the Great Pyramid. The temple-court is 74 cubits wide, and $\frac{5}{4} \times 74$ cubits from the east facade.

6400 palms - the use of the palm being predominant as we will see.

The computed bearing being $14^{\circ} 4'$ south of due east, or very close to the angle of $14^{\circ} 2'$ for an offset of one part southwards in four parts eastwards, we can now construct the plan of the causeway as shown in fig. 1. Four squares of side 1600 palms are placed from east to west to define the length and direction of the causeway itself, adjoining the side of a large square of 4×1600 equals 6400 palms to define the distance from the causeway to the north boundary of the plan, or north base of the Great Pyramid. The lower or eastern end of the causeway is placed $(6400 + 1600)$ or just 8000 palms southwards from the north boundary - a distance of 1142.85 cubits, in agreement with the data in Table I.

Table I **Coordinates of Causeway and Sphinx**

	In Survey Grid		In Ground Plan from N.E. Great Pyr.			
	Metres		Metres		Cubits	
	South	East	South	East	South	East
W. Causeway	82.4	134.5	478.74	-221.35	914.1	-422.6
E. Causeway	197.2	613.1	598.4	256.1	1142.5	489.0
W. Sphinx	152.7	531.6	553.1	175.0	1056.0	334.1
E. Sphinx	152.4	603.9	553.5	247.3	1056.8	472.2

(One cubit = 0.52375 ms. Bearing of ground plan to grid = $-35'$)

The Placing of the Sphinx

Supposing the use of round hundreds of palms in the design of the causeway to have been adopted also in positioning the Sphinx, the computed distance of about 1056 to 1057 cubits northwards from the axis of the Sphinx to the north boundary of the plan is seen to be 1057.14 cubits, or 7400 palms. The axis is then 600 palms northwards from the lower end of the causeway and 1000 palms southwards from the upper end. Although the accuracy of this placing should be established by further survey, an error of one cubit would not be excessive considering the distances involved; and the position is very probable since the dimension of 7400 palms is produced in a logical geometrical development of the ground plan. A factor of 74 is also found in dimensions in the three pyramids, and in the

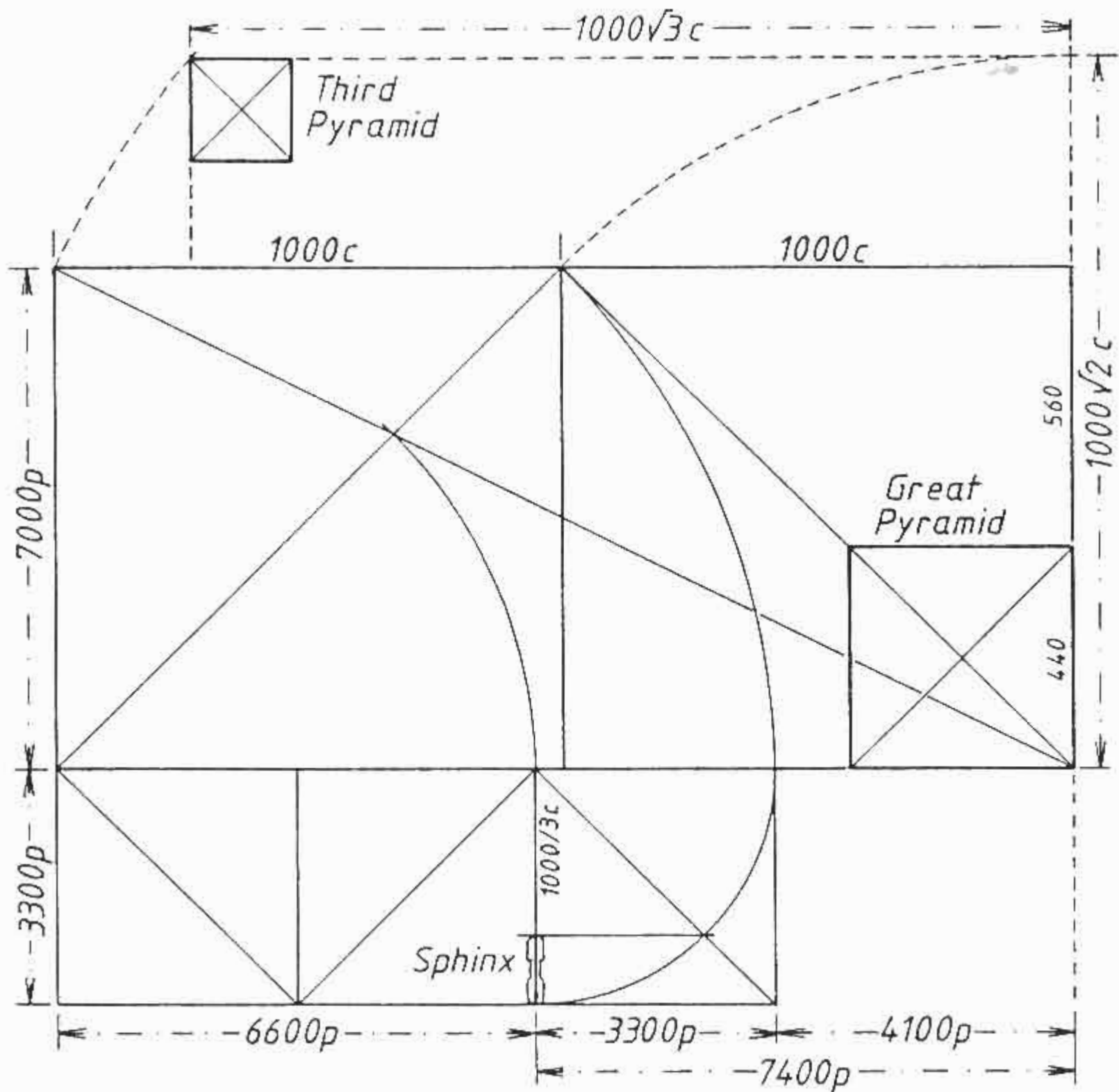


Figure 2. Geometrical Development for the Sphinx and Overall Dimensions of the Ground Plan.

Dimensions in royal cubits (c) and palms (p), developing from two squares of side 1000 cubits = 7000 palms.

design of the Upper Temple (see figure 1).

We have already shown that the overall dimensions of the plan can be developed by first taking the diagonal in a square of 1000 cubits to give an east-west dimension of $1000\sqrt{2}$ cubits, and next the diagonal in a rectangle of 1000 by $1000\sqrt{2}$ cubits to give the north-south dimension of $1000\sqrt{3}$ cubits. An equivalent geometry based on two squares of 1000 cubits is shown in figure 2. These squares are analogous to the two squares of 10 cubits in the floor plan of the King's Chamber in the Great Pyramid, where the height is equal to one-half the diagonal of the floor or $5\sqrt{5}$ cubits [9]. Since the diagonal of the ground plan is also the diagonal in a rectangle measuring 1000 by 2000 cubits, that is $1000\sqrt{5}$ cubits, the overall dimensions can be constructed as illustrated.

In referring to these diagonal measurements, it need not be assumed that the architect knew Pythagoras' theorem, since the required dimensions could have been estimated by direct measurement in a plan drawn to a suitable scale. In particular, the diagonal in a square of 10 cubits would be found to measure just 99 palms, that is 14.1428... cubits, with an error of less than 0.001 cubit. It is just this result that is shown in the placing of the Sphinx.

Marking off the diagonal of one square of 1000 cubits onto the side of the rectangle of 2000 cubits or 14000 palms, the latter is divided into parts of 9900 palms and 4100 palms. The smaller part recalls the round-figure base of the Second Pyramid of 410 cubits, and is analogous to the King's Chamber level in the Great Pyramid of $410/5$ equals 82 cubits, which derives from essentially the same geometry [10]. Dividing the diagonal of 9900 palms into thirds, as already given by the intersection of lines in fig. 2, the axis of the Sphinx is now placed $(4100 + 3300)$ or 7400 palms southwards from the north base of the Great Pyramid.

The validity of this construction is shown by its immediate continuation to give the distance eastwards from the Great Pyramid to the front of the Sphinx, which is now seen to be 3300 palms or 471.428 cubits; and also the distance to the rear of the Sphinx which is found to be the side of a square with a diagonal of 3300 palms, this being $1/3 \times 1000$ cubits. The length of the Sphinx is consequently defined as $1000(\sqrt{2} - 1)/3$ equals 138.07 or just 138 cubits [11], in close agreement with the survey plan.

Conclusions

Although the object of the pyramid-builders in placing the Sphinx within the ground plan of the Giza Pyramids may well be disputed, the simplicity with which the geometry of the plan is developed should leave little doubt as to the intention, especially since the resulting dimensional factors are reflected in the plans of the associated pyramids and temples. Contrary to the popular view that the Sphinx represented Khaefre in the role of the sun-god, Stadelmann has argued that the Sphinx must date back to the time of Khufu [12].

In the construction of the Giza complex of pyramids, temples and Sphinx in a unified plan over many decades may perhaps be seen the working of some hidden purpose, which diverted the resources of the state to its own ends. The impetus for the work possibly resided with the priests of Heliopolis, who would have devised the plans and may have had vested interests in seeing the project through to its completion. The diversity of the layouts employed in the temples must imply some differences of purpose, for which the outward funerary role was perhaps only the concealment.

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Notes

1. Legon, J.A.R., Discussions in Egyptology 10 (1988), 33-40.
2. As shown in [1], the elements are 250 cubits and $(440 \times 3/2 + 1)$ cubits from N. to S., and $(250 \times 5/2 - 1)$ cubits from E. to W.
3. Reisner, G.A., A History of the Giza Necropolis, Vol.1 (1942).
4. Lauer, J-Ph., A.S.A.E. XLVI (1947), 246.
5. Hassan, S., Excavations at Giza, Vol.IX (Cairo, 1960).
6. Hölscher, U. Das Grabdenkmal des Königs Chephren (Leipzig, 1912).
7. Ricke, H. Der Harmachistempel des Chefren (Beiträge Bf.10, 1970).
8. See Petrie's survey-data in [1] above.
9. Lauer, J-Ph, Le mystère des pyramides (Paris, 1974), 311.
10. Legon, J.A.R., Discussions in Egyptology 12 (1988) 41-48, fig.1.
11. This is 1/5 of the north-south distance of 690 cubits between the north sides of the Second and Great Pyramids.
12. Stadelmann, R., Die ägyptischen Pyramiden (Mainz, 1985), 125.